Proof of theorem 3.8:

$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

$(\hat{x}, \hat{y})^T = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} (x, y)^T = (\cos\theta\, x - \sin\theta\, y, \sin\theta\, x + \cos\theta\, y)^T$

$\cos\theta = c, \sin\theta = s$

$A(cx - sy)^2 + B(cx - sy)(sx + cy) + C(sx + cy)^2 + Things\ that\ do\ not\ affect\ discriminant = 0$

$Ac^2x^2 - 2Acsxy + As^2y^2 + Bcsx^2 + Bc^2xy - Bs^2xy - Bcsy^2 + Cs^2x^2 + 2Ccsxy + Cc^2y^2 + \cdots = 0$

$Ac^2x^2 + Bcsx^2 + Cs^2x^2 - 2Acsxy + Bc^2xy - Bs^2xy + 2Ccsxy + As^2y^2 - Bcsy^2 + Cc^2y^2 + \cdots = 0$

$(Ac^2 + Bcs + Cs^2)x^2 + (-2Acs + Bc^2 - Bs^2 + 2Ccs)xy + (As^2 - Bcs + Cc^2)y^2 + \cdots = 0$

Discriminant:

$(-2Acs + Bc^2 - Bs^2 + 2Ccs)^2 - 4(Ac^2 + Bcs + Cs^2)(As^2 - Bcs + Cc^2) =$

$4AcBs^3 - 2c^2B^2s^2 + B^2s^4 - 4cBCs^3 + 4A^2c^2s^2 - 4Ac^3Bs - 8Ac^2Cs^2 + 4c^3BCs + 4c^2C^2s^2 + c^4B^2 +$

$-4A^2c^2s^2 + 4Ac^3Bs - 4Ac^4C - 4AcBs^3 + 4c^2B^2s^2 - 4c^3BCs - 4ACs^4 + 4cBCs^3 - 4c^2C^2s^2 =$

$2c^2B^2s^2 + B^2s^4 - 8Ac^2Cs^2 + c^4B^2 - 4Ac^4C - 4ACs^4 =$

$B^2(s^4 + 2c^2s^2 + c^4) - 4AC(s^4 + 2c^2s^2 + c^4) = B^2\left(\underbrace{s^2 + c^2}_{\sin^2\theta + \cos^2\theta}\right)^2 - 4AC(s^2 + c^2)^2 = B^2 - 4AC$