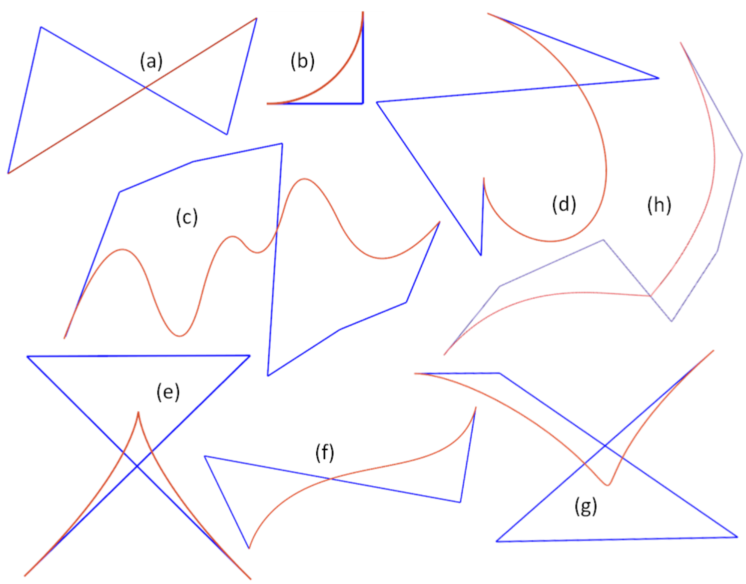
236716
Computer Aided Geometric Design
HW 1

Instructor: Gershon Elber
T.A.: Boris van Sosin

Handed Out: May 21th, 2020 Due Date: May 22th, 2020 (8:30)

1. (20pt) According to the fundamental theorem of differential geometry, a planar curve (torsion is zero) is uniquely determined (up to its position and orientation) if its curvature, $\kappa(s)$, is known. Consider a starting position P and Frenet frame $\{T(0), N(0), B(0)\}$ and the scalar univariate function, $\kappa(s)$, $s \in [0, L]$, arc length parametrized. Propose an algorithm which plots the 2D curve. How, if at all, your answer will change if κ is paramterized using a general regular parameterization $\kappa(t)$?
2. (20pt) Which of the following curves can or cannot be a polynomial and/or rational Bézier curve:



   In blue, the control polygon is shown and in red the curve itself. Explain your answer in a sentence.
3. (20pt).
   (a) Prove or provide a counter example: if the control polygon of a polynomial Bézier curve $y(t)$ is monotone (not-monotone) in y, $y(t)$ is monotone (not-monotone) as well.
   (b) Repeat 1 for a rational Bézier curve. Did you make any assumptions on the values of the weights?
4. (20pt). Let $c(t) = (c_x(t), c_y(t))$ be a planar parametric curve, and let $\kappa(t)$ be its curvature field.
   (a) Transformations of the forms $(c_x(t), c_y(t)) + (T_x, T_y)$, and $\begin{pmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{pmatrix} \begin{pmatrix} c_x(t) \\ c_y(t) \end{pmatrix}$ apply translation by vector (T_x, T_y), and rotation by an angle θ (respectively) on planar curves. Prove that the curvature of $c(t)$ is preserved under translation and rotation.
   (b) A matrix of the form $\begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}$ applies scaling by a factor of λ on planar curves. Compute the curvature of $\begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} \begin{pmatrix} c_x(t) \\ c_y(t) \end{pmatrix}$.
   (c) Explain in qualitative terms why the above results make sense geometrically.
5. (20pt). Let $C(t) = (x(t), y(t))$ be a regular planar curve, and let $C_{o_d}(t) = C(t) + N(t)d$ be the offset curve of $C(t)$ with amount d, $N(t)$ being the unit normal of C.
   Is C_{o_d} always regular? If so explain. If not, under what conditions will C_{o_d} be singular?