236716
Computer Aided Geometric Design
HW 2
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Handed Out: July 2nd, 2020 Due Date: 8:00, July 3rd, 2020

1. (20pt) Prove or dispute:
   (a) If all the control points of a B-spline curve are on some B-spline surface, then all the curve must be contained in the surface.
   (b) If all the control points of a linear B-spline curve are on some B-spline surface, then all the curve must be contained in the surface.
   (c) If all the control points of a linear B-spline curve are on some bi-linear B-spline surface, then all the curve must be contained in the surface.

2. (20pt)
   (a) For a torus surface, what are the locations where the Gaussian curvature vanish? Justify your answer.
   (b) What are the locations where the Gaussian curvature vanish on a sufficiently continuous surface of revolution, if any. Justify your answer.
   (c) Prove or dispute: a ruled surface can never be elliptic (will never possess a positive Gaussian curvature).

3. (25pt) Let $\alpha(t)$ be an arc-length parameterized curve. In this question, we will denote a derivative curve of some curve as its Hodograph.
   (a) What is the shape (or locus of points) of the Hodograph of $\alpha(t)$? What is the curvature of the Hodograph?
   (b) Let $\beta(t)$ be a B-spline curve with a knot vector $\boldsymbol{\tau} = (\tau_0, ..., \tau_{n-1})$. Let $D^\beta = [\tau_{min}, \tau_{max}]$ be the parametric domain of $\beta(t)$. Assume there is a black-box algorithm, $\mathcal{L}$, such that $\mathcal{L}(\beta) = L$ is the length of a curve $\beta(t)$ in D^β.

      Given a curve β of length L, propose a method for producing a curve γ, which has the same shape as β, and traverses the length of $\beta(t)$ in time L.
   (c) We would like to use the above to construct a piecewise ϵ-approximate arc-length paramteric curve. A regular paramteric curve γ is piecewise ϵ-approximate arc-length if throughout its domain, it satisfies

$$1 - \epsilon \leq |\gamma'(r)| \leq 1 + \epsilon.$$

      Propose an algorithm, that given a regular B-spline curve of finite length, $\beta(t)$, construct a piecewise ϵ-approximate arc-length curve γ with the same shape. Assume there is a way to verify that $1 - \epsilon \leq |\gamma'(r)| \leq 1 + \epsilon$. What is the continuity of your γ? What is the shape of the hodograph of γ?

4. (15pt) In surface-surface intersections, bounds on the normals of a surface play a major role. Given a regular parametric surface S, can you propose a (tight as possible) algorithm to bound all possible normal direction assumed by S? A hint: start with a shape that can bound (directions of) vector fields (i.e. the normal field of S).

5. (20pt) One of the most famous Appollonius problems is about finding a circle that is tangent to three given circles, in the plane, and in general position (i.e. no tangencies, nor singularities, etc. are in the three circles).

   However, herein, we can apply this question to planar curves, in general position: Given three non-interesecting regular parametric planar curves, $C_i(t)$, $i = 1, 2, 3$, derive the necessary algebraic constraints to the problem (so the presented multivariate solver can solve them!). Be clear as to the number of degrees of freedom, and who they are, and the number of constraints you have.

   Do we always have at least one solution for three non-interesecting regular parametric planar curves?