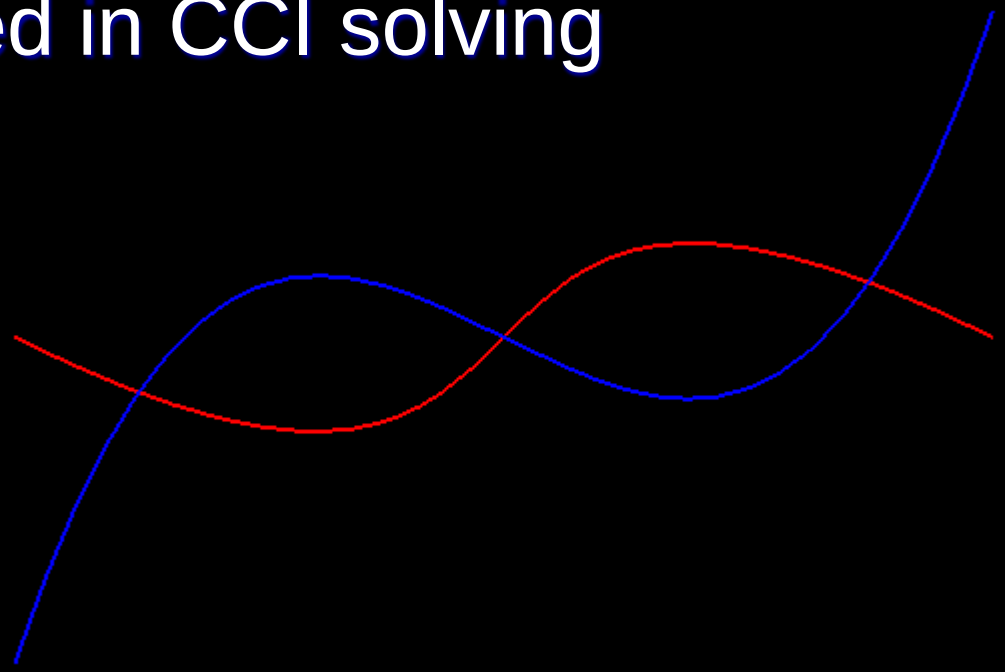
Computer Aided Geometric Design

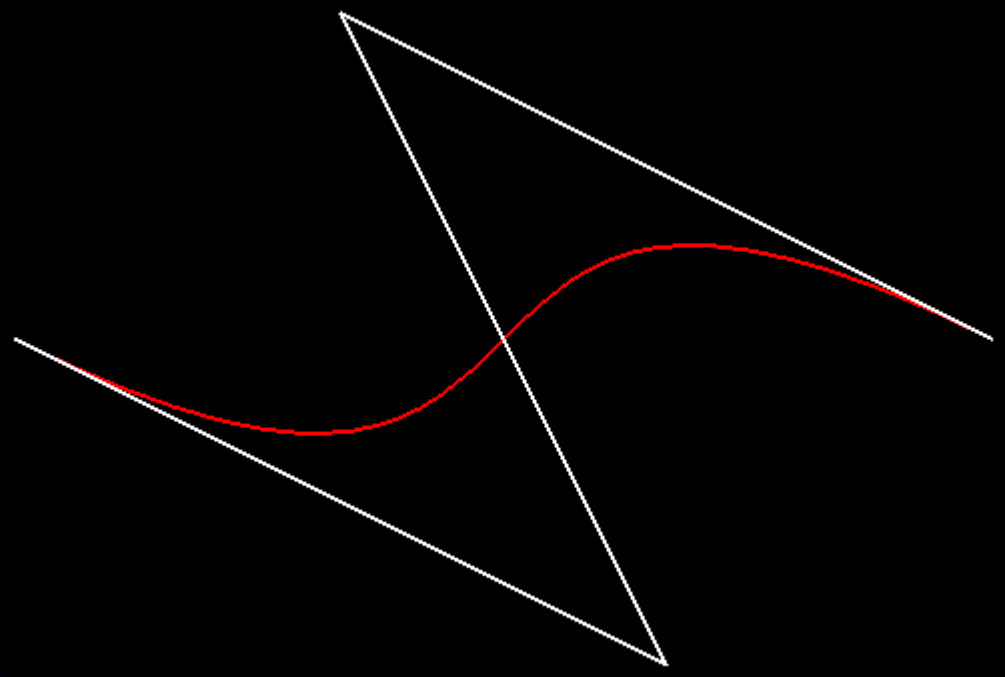
The Bézier curve-curve intersection problem

- How the geometric properties of Bézier curves are used in CCI solving



The convex hull property

Recall: a Bézier curve is contained inside the convex hull of its control mesh



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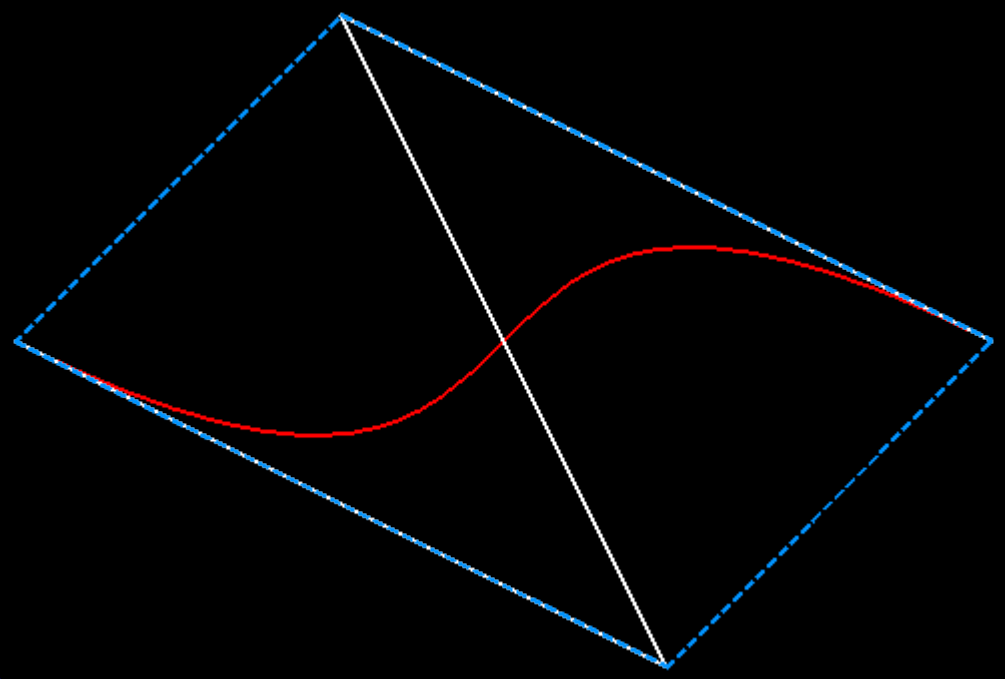
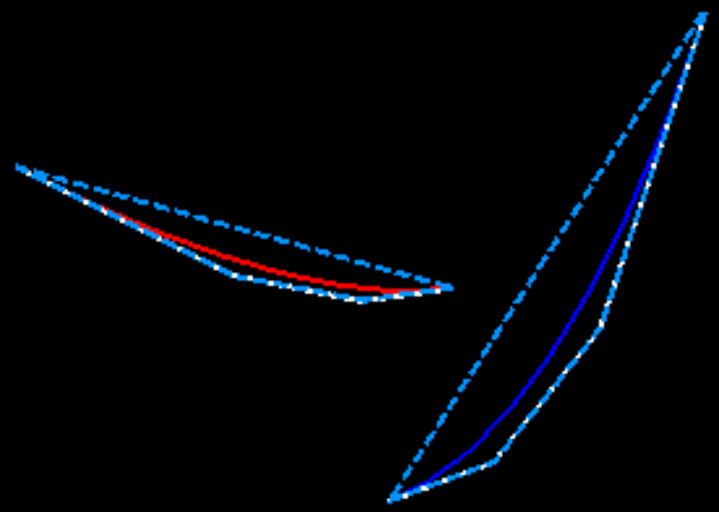


The convex hull property

Useful for non-intersection testing:

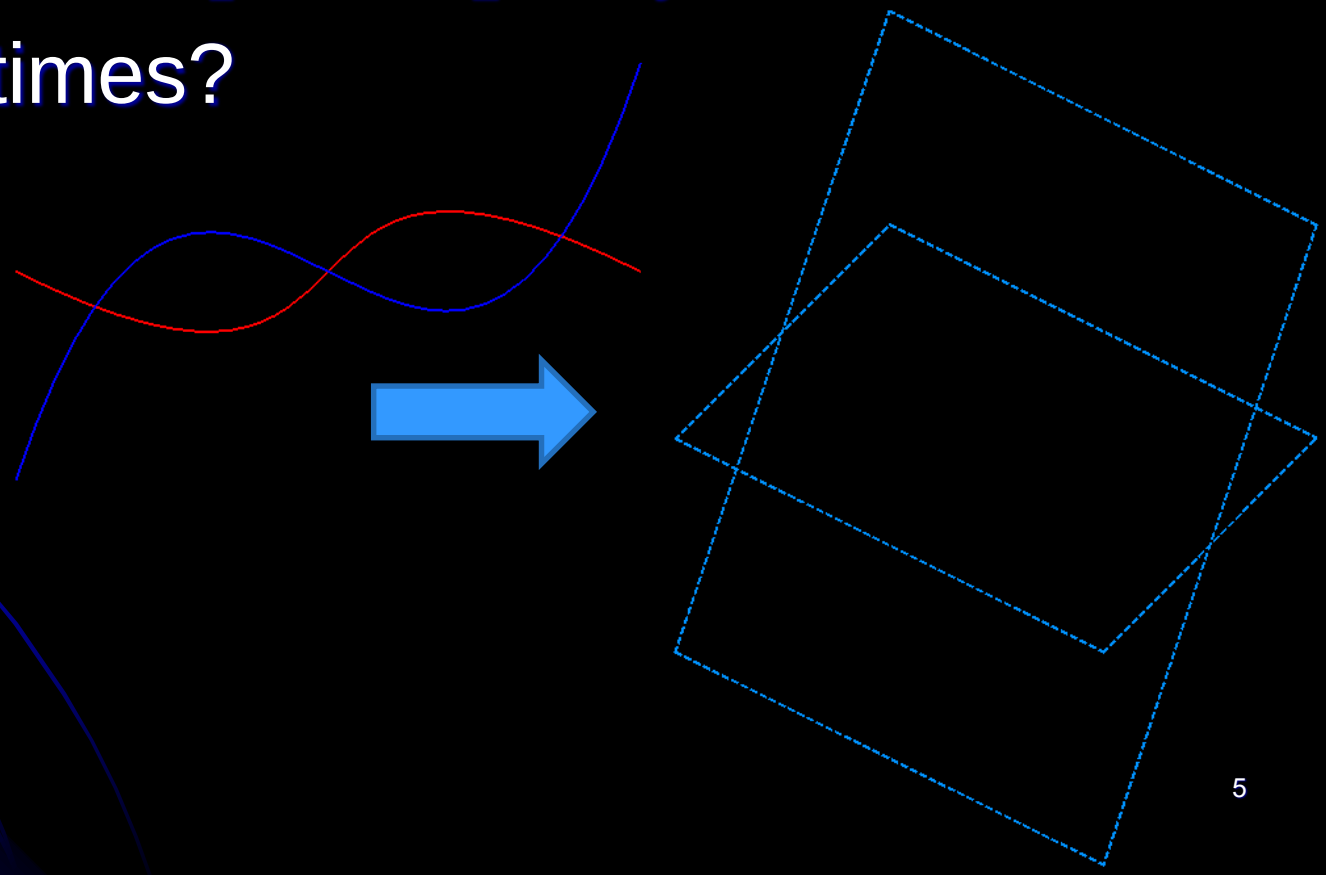
For curves $c_1(t), c_2(r)$, if the convex hulls of control meshes of c_1 and c_2 are disjoint, then c_1 and c_2 do not intersect.



The convex hull property

If the convex hulls of the control meshes do intersect then c_1 and c_2 may intersect.

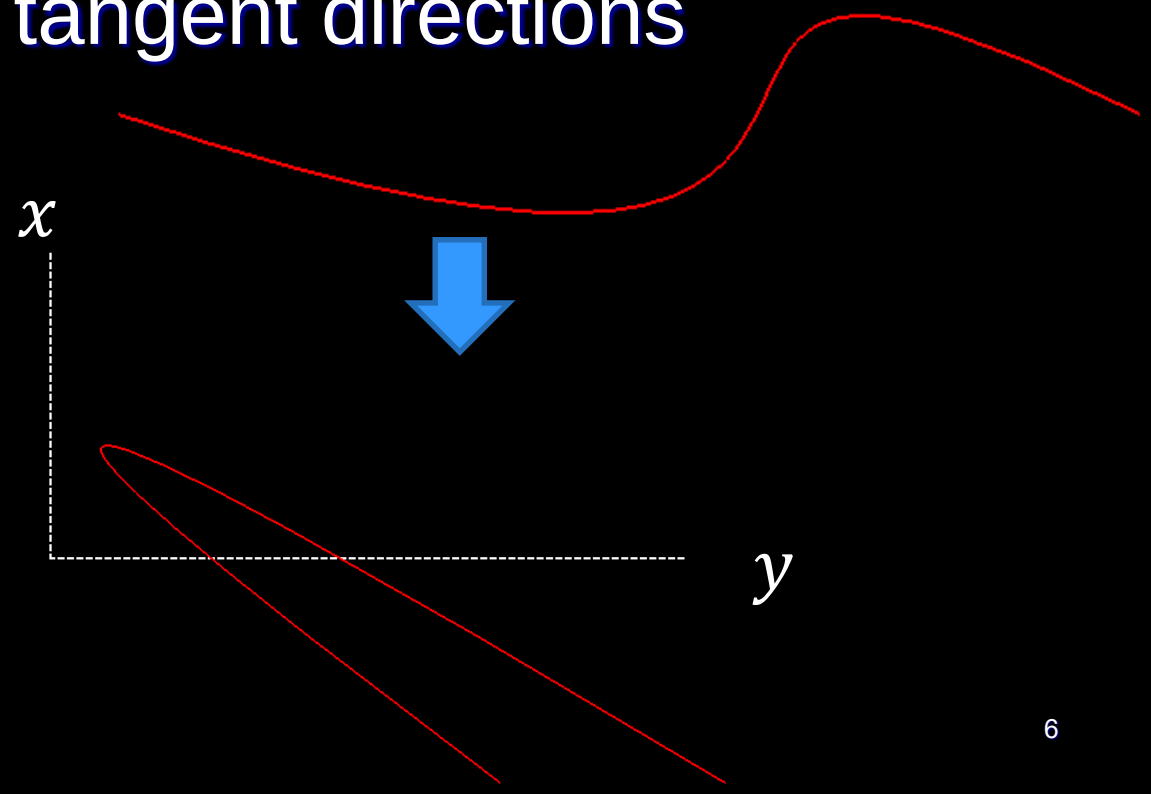
How many times?



Testing for multiple intersections

The hodograph of $c(t)$ is the curve of tangent directions of c. Equals to $c'(t)$.

Bound the set of tangent directions in a pair of cones.



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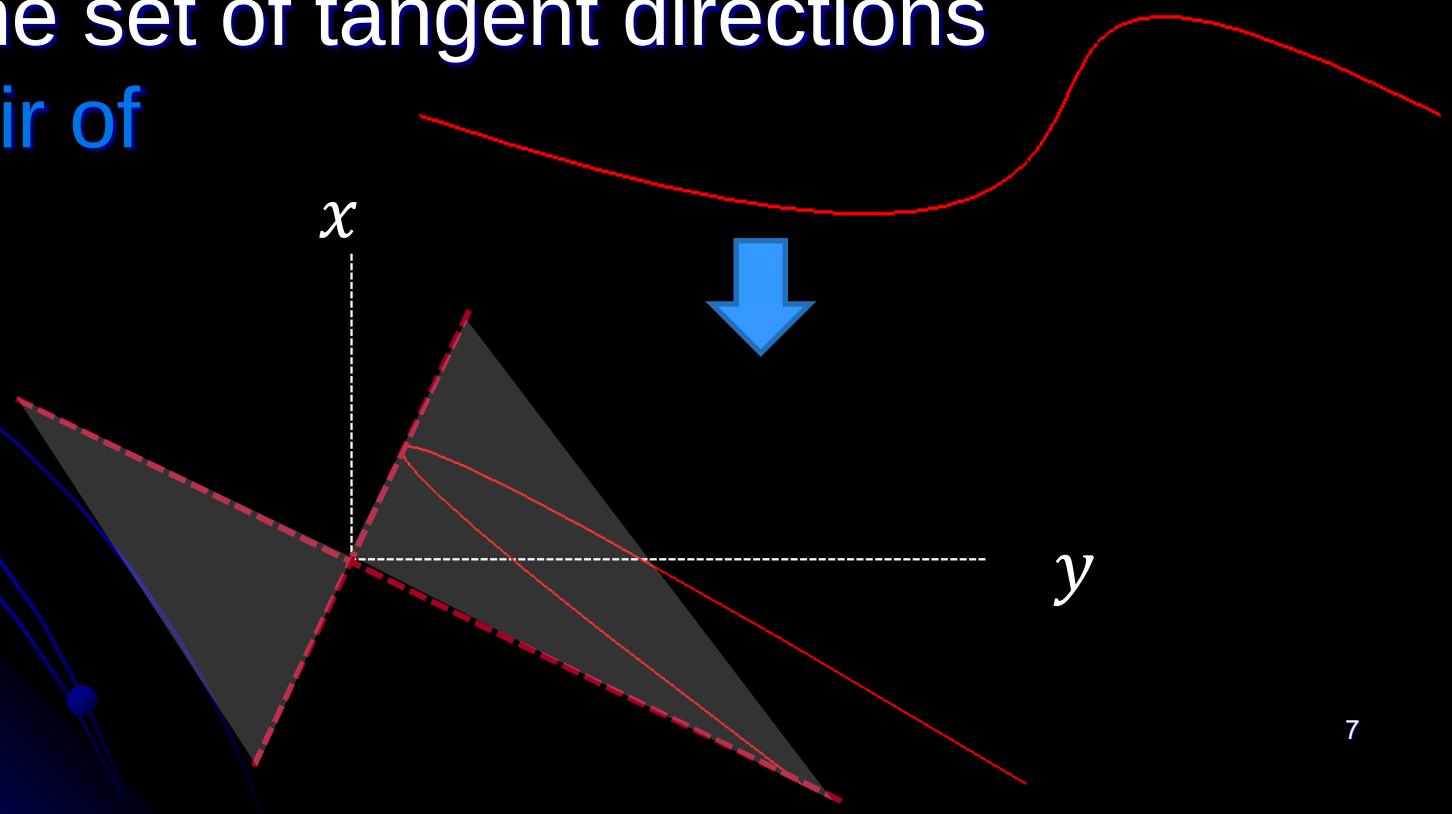

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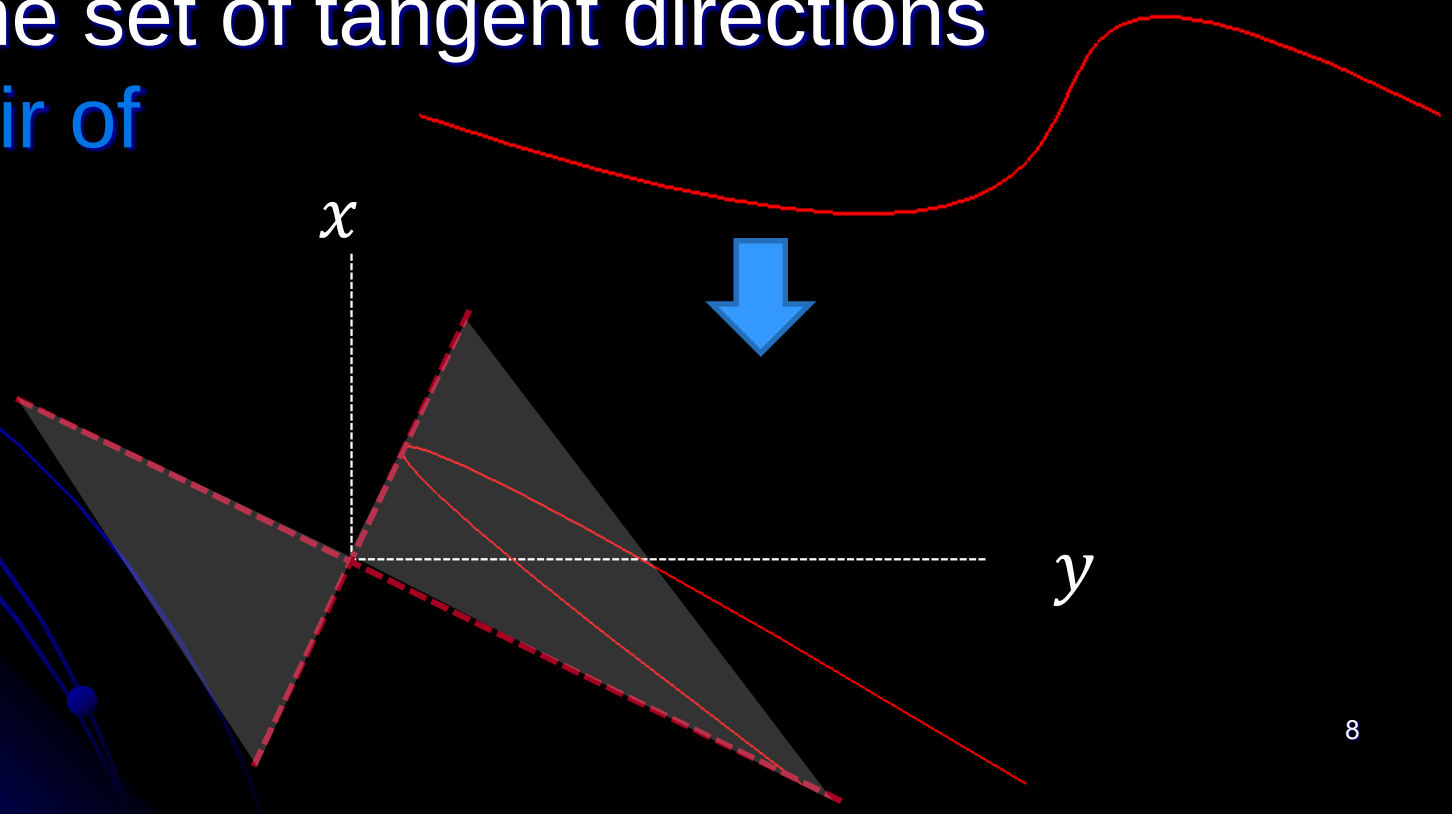


Testing for multiple intersections

If we position the pair of cones on any point on $c(t)$, the entire curve will be contained in the cones.

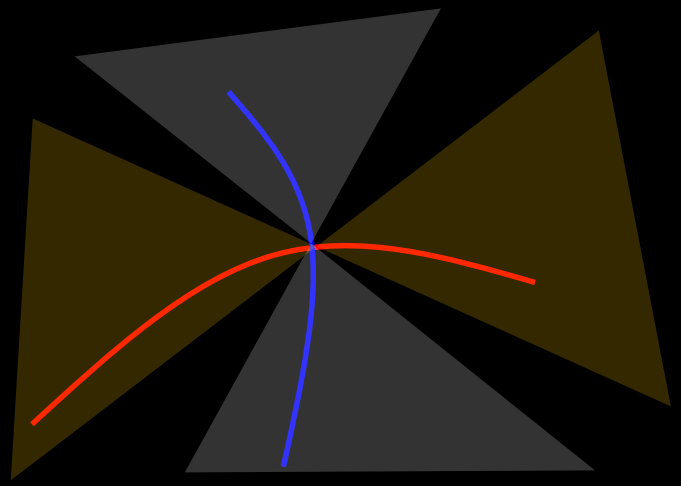
Proof: Position cones at $c(t_0)$ and assume (falsely) $c(t_1)$ is outside the cones.

- There is t_2 such that $\pm c'(t_2) = c(t_1) - c(t_0)$.

But the cones where constructed such that they contain all of $c'(t)$ (including $c'(t_2)$). This is a contradiction. ■

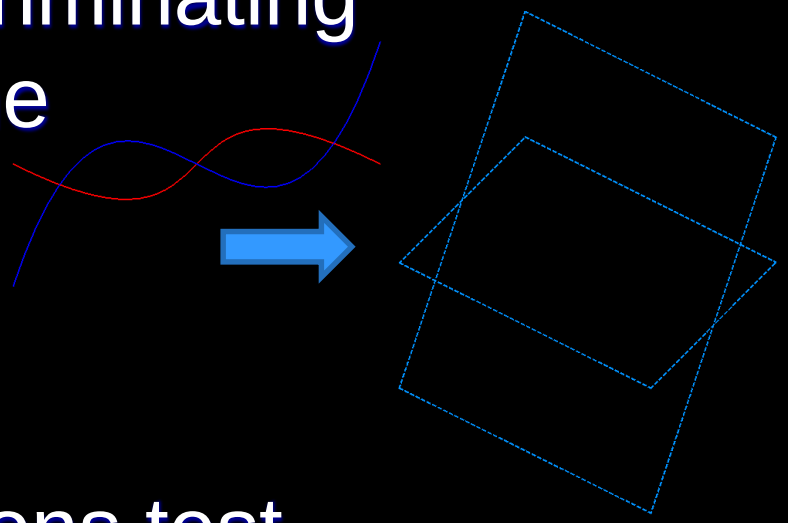
Testing for multiple intersections

According to this theorem,
if the cones of tangent directions of c_0 and c_1 do not overlap (except for the apex),
then c_0 and c_1 intersect at most once.

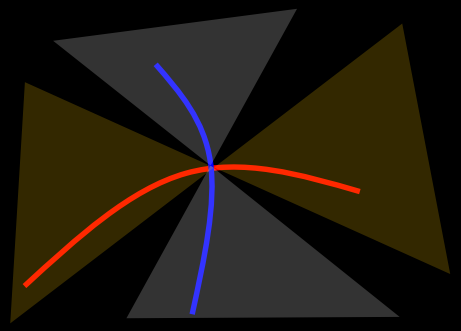
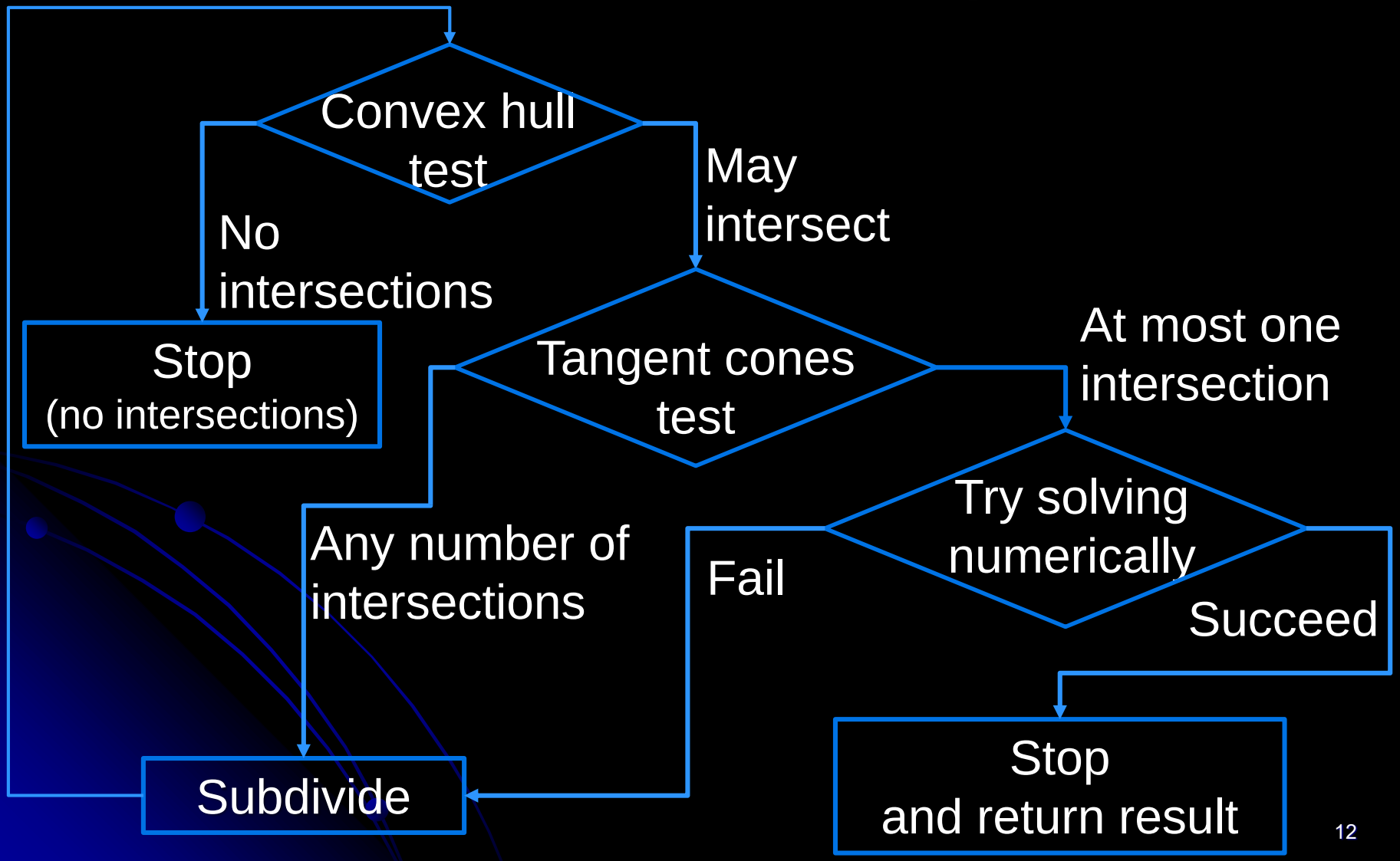


Intermediate summary

- Convex hull test for discriminating between cases where the curves have
  no intersections vs.
  may have intersections.



- Cones of tangent directions test for discriminating between cases where the curves have
  at most one intersection vs.
  may any number of intersections.



CCI solution complete algorithm