Bézier 2 B-spline

• A single Bézier curve can clearly be represented as a single Bspline curve

• Can a single B-spline curve be represented as a single Bézier curve? Always? Sometimes? Under what conditions?

• Not always. B-spline curves are polynomials in parts. They posses finite continuity at the knots whereas Bézier curves have infinite continuity.

• Any B-spline curve that has the support domain with no interior knots can be represented as a simple Bézier curve
Bézier 2 B-spline

- A single Bézier curve can clearly be represented as a single B-spline curve.
- Can a single B-spline curve be represented as several Bézier curves? Always? Sometimes? Under what conditions?
- Yes.
- Assume the interior knots of \( \gamma(t) = \sum_{i=0}^{n} P_i B_{i,k}(t) \) are \( \tau = (\tau_0, \tau_1, \ldots, \tau_m) \) with multiplicities \( \mu_0, \mu_1, \ldots, \mu_m \). Assume open end conditions.
Bézier 2 B-spline

- We perform knots insertion of each knot \( \tau_i \) of which its multiplicity is less than \( k \), by inserting new knots of \( k+1 \) consecutive control points as a control polygon at a new Bézier curve.
- Question: What if the B-spline curve is not open end?
Bézier 2 B-spline

• Let \( C(t) = \sum_{i=0}^{2} P_i B_{i,2}(t) \) be a quadratic Bézier curve.

• What would be the control points of the two curves \( C_1(t) = \sum_{i=0}^{2} Q_i B_{i,3}(t) \) and \( C_2(t) = \sum_{i=0}^{2} R_i B_{i,3}(t) \) for \( C(t) \) subdivided at 1/3?

• Assume \( C(t) = D(t) = \sum_{i=0}^{2} P_i B_{i,3,\tau}(t) \) are defined over the knot sequence \( \tau = \{0,0,0,1,1,1\} \). Show the new control points of \( D(t) \) after a single knot insertion of knot 1/3, after a double knot insertion of 1/3, and after a triple knot insertion of 1/3. What is the connection to the subdivision of the Bézier curve in the previous section?
Bézier 2 B-spline

- Let $C(t) = \sum_{i=0}^{2} P_i B_{i,2}(t)$ be a quadratic Bézier curve.

- What would be the control points of the two curves $C_1(t) = \sum_{i=3}^{2} Q_i B_{i,2}(t)$ and $C_2(t) = \sum_{i=0}^{2} R_i B_{i,3}(t)$ for $C(t)$ subdivided at $1/3$?

  - $Q_0 = P_0$
  - $Q_1 = \frac{2P_0 + P_1}{3}$
  - $Q_2 = \frac{4P_0 + 4P_1 + P_2}{9}$

  - $R_0 = \frac{4P_0 + 4P_1 + P_2}{9}$
  - $R_1 = \frac{2P_1 + P_2}{3}$
  - $R_2 = P_2$
Bézier 2 B-spline

Let \( C(t) = \sum_{i=0}^{2} P_i B_{i,2}(t) \) be a quadratic Bézier curve.

Assume \( C(t) = D(t) = \sum_{i=0}^{2} P_i B_{i,3,(\tau(t))} \) are defined over the knot sequence \( \tau = \{0,0,0,1,1,1\} \). Show the new control points of \( D(t) \) after a single knot insertion of knot \( 1/3 \), after a double knot insertion of \( 1/3 \), and after a triple knot insertion of \( 1/3 \). What is the connection to the subdivision of the Bézier curve in the previous section?
Bézier 2 B-spline

\[
\begin{align*}
Q_0 &= P_0 \\
Q_1 &= \frac{2P_0 + P_1}{3} \\
Q_2 &= \frac{2P_1 + P_2}{3} \\
Q_3 &= P_2 \\
R_0 &= P_0 \\
R_1 &= \frac{2P_0 + P_1}{3} \\
R_2 &= \frac{4P_0 + 4P_1 + P_2}{9} \\
R_3 &= \frac{2P_1 + P_2}{3} \\
R_4 &= P_2 \\
S_0 &= P_0 \\
S_1 &= \frac{2P_0 + P_1}{3} \\
S_2 &= \frac{4P_0 + 4P_1 + P_2}{9} \\
S_3 &= \frac{4P_0 + 4P_1 + P_2}{9} \\
S_4 &= \frac{2P_1 + P_2}{3} \\
S_5 &= P_2
\end{align*}
\]

- Although the multiplicity of \(1/3\) is 3 there is one double point \(S_2 = S_3\).
Bézier 2 B-spline