- A single Bézier curve can clearly be represented as a single Bspline curve
- Can a single B-spline curve be represented as a single Bézier curve ? Always ? Sometimes ? Under what conditions ?
- Not always. B-spline curves are polynomials in parts. They posses finite continuity at the knots whereas Bézier curves have infinite continuity.
- Any B-spline curve that has the support domain with no interior knots can be represented as a simple Bézier curve

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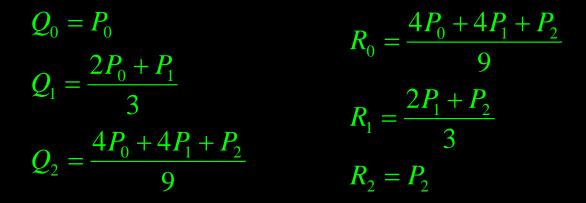
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- A single Bézier curve can clearly be represented as a single Bspline curve
- Can a single Bspline curve be represented as several Bézier curve ? Always ? Sometimes ? Under what conditions ?
- Yes.
- Assume the interior knots of $\gamma(t) = \sum_{i=0}^{n} P_i B_{i,k}(t)$ are $\tau = (\tau_0, \tau_1, \dots, \tau_m)$ with multiplicities $\mu_0, \mu_1, \dots, \mu_m$. Assume open end conditions.

- We perform knots insertion of each knot τ_i of which its multiplicity is less then k, by inserting new knots of k+1 consecutive control points as a control polygon at a new Bézier curve .
- Question: What if the B-spline curve is not open end ?

- Let $C(t) = \sum_{i=0}^{2} P_i B_{i,2}(t)$ be a quadratic Bézier curve.
- What would be the control points of the two curves $C_1(t) = \sum_{i=0}^2 Q_i B_{i,3}(t)$ and $C_2(t) = \sum_{i=0}^2 R_i B_{i,3}(t)$ for C(t) subdivided at 1/3 ?
- Assume $C(t) = D(t) = \sum_{i=0}^{2} P_i B_{i,3,\tau}(t)$ are defined over the knot sequence $\tau = \{0,0,0,1,1,1\}$. Show the new control points of D(t) after a single knot insertion of knot 1/3, after a double knot insertion of 1/3, and after a triple knot insertion of 1/3. What is the connection to the subdivision of the Bézier curve in the previous section?

- Let $C(t) = \sum_{i=0}^{2} P_i B_{i,2}(t)$ be a quadratic Bézier curve.
- What would be the control points of the two curves $C_1(t) = \sum_{i=3}^2 Q_i B_{i,2}(t)$ and $C_2(t) = \sum_{i=0}^2 R_i B_{i,3}(t)$ for C(t) subdivided at 1/3 ?



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- Let $C(t) = \sum_{i=0}^{2} P_i B_{i,2}(t)$ be a quadratic Bézier curve.
- Assume $C(t) = D(t) = \sum_{i=0}^{2} P_i B_{i,3,\tau}(t)$ are defined over the knot sequence $\tau = \{0,0,0,1,1,1\}$. Show the new control points of D(t) after a single knot insertion of knot 1/3, after a double knot insertion of 1/3, and after a triple knot insertion of 1/3. What is the connection to the subdivision of the Bézier curve in the previous section?

Bézier 2 B-spline

| $Q_0 = P_0$ | $R_0 = P_0$ | $S_0 = P_0$ |
|------------------------------|-------------------------------------|-------------------------------------|
| $Q_1 = \frac{2P_0 + P_1}{3}$ | $R_1 = \frac{2P_0 + P_1}{3}$ | $S_1 = \frac{2P_0 + P_1}{3}$ |
| $Q_2 = \frac{2P_1 + P_2}{3}$ | $R_2 = \frac{4P_0 + 4P_1 + P_2}{9}$ | $S_2 = \frac{4P_0 + 4P_1 + P_2}{9}$ |
| $Q_3 = P_2$ | $R_3 = \frac{2P_1 + P_2}{3}$ | $S_3 = \frac{4P_0 + 4P_1 + P_2}{9}$ |
| | $R_4 = P_2$ | $S_4 = \frac{2P_1 + P_2}{3}$ |
| | | $S_z = P_z$ |

• Although the multiplicity of 1/3 is 3 there is one double point $S_2 = S_3$.

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