Bézier 2 B-spline

- A single Bézier curve can clearly be represented as a single Bspline curve
- Can a single B-spline curve be represented as a single Bézier curve ? Always ? Sometimes ? Under what conditions ?

• Not always. B-spline curves are polynomials in parts. They posses finite continuity at the knots whereas Bézier curves have infinite continuity.

• Any B-spline curve that has the support domain with no interior knots can be represented as a simple Bézier curve

Bézier 2 B-spline

- A single Bézier curve can clearly be represented as a single Bspline curve
- Can a single Bspline curve be represented as several Bézier curve ? Always ? Sometimes ? Under what conditions ?

• Yes.
• Assume the interior knots of $\gamma(t) = \sum_{i=0}^{n} P_i B_{i,k}(t)$ are $\boldsymbol{\tau} = (\tau_0, \tau_1, \ldots, \tau_m)$ with multiplicities $\mu_0, \mu_1, \ldots, \mu_m$. Assume open end conditions.

Bézier 2 B-spline

- We perform knots insertion of each knot τ_i of which its multiplicity is less then k, by inserting new knots of $k+1$ consecutive control points as a control polygon at a new Bézier curve .
- Question: What if the B-spline curve is not open end ?

Bézier 2 B-spline

- Let $C(t)=\sum_{i=0}^{2} P_i B_{i,2}(t)$ be a quadratic Bézier curve.

- What would be the control points of the two curves $C_1(t)=\sum_{i=0}^{2} Q_i B_{i,3}(t)$ and $C_2(t)=\sum_{i=0}^{2} R_i B_{i,3}(t)$ for $C(t)$ subdivided at 1/3 ?

- Assume $C(t)=D(t)=\sum_{i=0}^{2} P_i B_{i,3,\tau}(t)$ are defined over the knot sequence $\tau=\{0,0,0,1,1,1\}$. Show the new control points of D(t) after a single knot insertion of knot 1/3, after a double knot insertion of 1/3, and after a triple knot insertion of 1/3. What is the connection to the subdivision of the Bézier curve in the previous section?

Bézier 2 B-spline

- Let $C(t) = \sum_{i=0}^{2} P_i B_{i,2}(t)$ be a quadratic Bézier curve.

- What would be the control points of the two curves $C_1(t) = \sum_{i=3}^{2} Q_i B_{i,2}(t)$ and $C_2(t) = \sum_{i=0}^{2} R_i B_{i,3}(t)$ for $C(t)$ subdivided at $1/3$?

$$Q_0 = P_0$$

$$Q_1 = \frac{2P_0 + P_1}{3}$$

$$Q_2 = \frac{4P_0 + 4P_1 + P_2}{9}$$

$$R_0 = \frac{4P_0 + 4P_1 + P_2}{9}$$

$$R_1 = \frac{2P_1 + P_2}{3}$$

$$R_2 = P_2$$

Bézier 2 B-spline

- Let $C(t) = \sum_{i=0}^{2} P_i B_{i,2}(t)$ be a quadratic Bézier curve.
- Assume $C(t) = D(t) = \sum_{i=0}^{2} P_i B_{i,3,\tau}(t)$ are defined over the knot sequence $\tau = \{0,0,0,1,1,1\}$. Show the new control points of D(t) after a single knot insertion of knot 1/3, after a double knot insertion of 1/3, and after a triple knot insertion of 1/3. What is the connection to the subdivision of the Bézier curve in the previous section?

Bézier 2 B-spline

$$Q_0 = P_0 \qquad R_0 = P_0 \qquad S_0 = P_0$$

$$Q_1 = \frac{2P_0 + P_1}{3} \qquad R_1 = \frac{2P_0 + P_1}{3} \qquad S_1 = \frac{2P_0 + P_1}{3}$$

$$Q_2 = \frac{2P_1 + P_2}{3} \qquad R_2 = \frac{4P_0 + 4P_1 + P_2}{9} \qquad S_2 = \frac{4P_0 + 4P_1 + P_2}{9}$$

$$Q_3 = P_2 \qquad R_3 = \frac{2P_1 + P_2}{3} \qquad S_3 = \frac{4P_0 + 4P_1 + P_2}{9}$$

$$R_4 = P_2 \qquad S_4 = \frac{2P_1 + P_2}{3}$$

$$S_5 = P_2$$

- Although the multiplicity of 1/3 is 3 there is one double point $S_2 = S_3$.

Bézier 2 B-spline