Problem Definition

- Chapter 2 Gershon Elber’s PhD Thesis: Symbolic and Numeric Computation

Bezier or

B-spline curve

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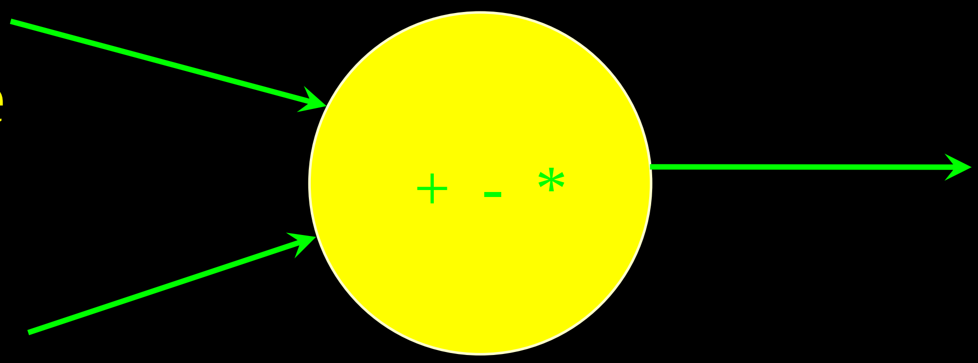


Bezier or

B-spline curve

Problem Definition

- What about division ?
- What is a B-spline curve ?

$$C(t) = \sum_{i=0}^{n} P_i B_{i,k,\tau}(t)$$

Sum and Difference

- Consider two B-spline curves

$$C_1(t) = \sum_{i=0}^{n} P_i B_{i,p,\tau}(t) \quad \text{and} \quad C_2(t) = \sum_{i=0}^{n} Q_i B_{i,q,\mu}(t)$$

  - If the parametric domains of $C_1(t)$ and $C_2(t)$ are different then employ an affine transform to build a common parametric domain
  - If ($p \neq q$) degree raise the lower degree curve until $p = q$
  - If ($\tau \neq \mu$) insert knots such that $\tau = \mu$
  - Finally,

$$C_2(t) \pm C_2(t) = \sum_{i=0}^{n} (P_i \pm Q_i) B_{i,p,\tau}(t)$$

Sum and Difference

- Consider a Bezier curve to be degree raised

$$C(t)=\sum_{i=0}^{n} P_i \theta_{i,k}(t)$$

$$(1-t)\theta_{i,p}(t)=\frac{n!}{i!(n-i)!}t^i(1-t)^{n-i+1}=\frac{n-i+1}{n+1}\theta_{i,p+1}(t)$$

$$t\theta_{i,p}(t)=\frac{n!}{i!(n-i)!}t^{i+1}(1-t)^{n-i}=\frac{i+1}{n+1}\theta_{i+1,p+1}(t)$$

$$\theta_{i,p}(t)=(1-t)\theta_{i,p}(t)+t\theta_{i,p}(t)=\frac{n-i+1}{n+1}\theta_{i,p+1}(t)+\frac{i+1}{n+1}\theta_{i+1,p+1}(t)$$

Sum and Difference

- Consider two Bezier curves

$$C_1(t)=\sum_{i=0}^{n} P_i \theta_{i,p}(t) \quad \text{and} \quad C_2(t)=\sum_{i=0}^{n} Q_i B_{i,q}(t)$$

  - If ($p \neq q$) degree raise the lower degree curve until $p = q$

$$(1-t)\theta_{i,p}(t)=\frac{n!}{i!(n-i)!}t^i(1-t)^{n-i+1}=\frac{n-i+1}{n+1}\theta_{i,p+1}(t)$$

$$t\theta_{i,p}(t)=\frac{n!}{i!(n-i)!}t^{i+1}(1-t)^{n-i}=\frac{i+1}{n+1}\theta_{i+1,p+1}(t)$$

$$\theta_{i,p}(t)=(1-t)\theta_{i,p}(t)+t\theta_{i,p}(t)=\frac{n-i+1}{n+1}\theta_{i,p+1}(t)+\frac{i+1}{n+1}\theta_{i+1,p+1}(t)$$

Product of B-spline Curves

- Reconsider two B-spline curves

$$C_1(t) = \sum_{i=0}^{n_1} P_i B_{i,p,\tau_1}(t) \quad \text{and} \quad C_2(t) = \sum_{i=0}^{n_2} Q_i B_{i,q,\tau_2}(t)$$

  - These curves have:
    - Distinct knot sequences: ν^1 and ν^2
    - Degrees: p and q
    - Multiplicities: $\mu^1(\lambda_j)$ and $\mu^2(\lambda_j)$

- Output:

$$C(t) = \sum_{i=0}^{n} P_i B_{i,p+q,\tau}(t)$$

Product of B-spline Curves

- Immediate inference: the degree of curve C is: $p+q$
- Continuity inferences:

  - c_1 has continuity bounded by $p-\mu^1(\lambda_j)$ at knot $\lambda_j \in \tau_1$
  - c_2 has continuity bounded by $q-\mu^2(\lambda_j)$ at knot $\lambda_j \in \tau_2$
  - Immediate conclusion: c has continuity bounded by

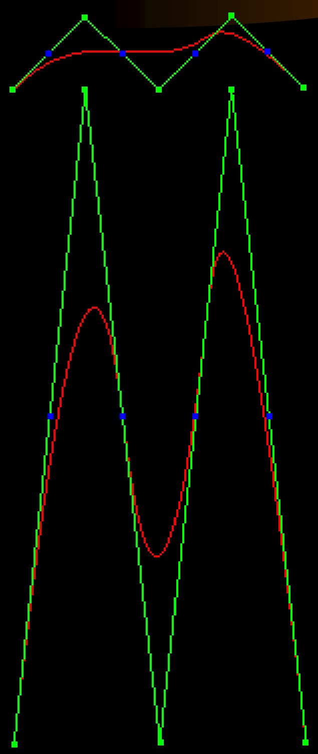
$$\min\left(q-\mu^1(\lambda_j), q-\mu^2(\lambda_j)\right)$$

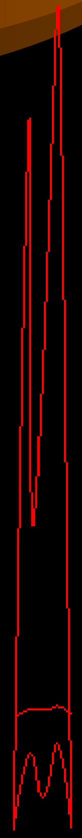
- Remark: We have the degree and the continuity of the resulting curve

Product of B-spline Curves - Example

- C_1 has:
  - ➢ Degree: 2
  - ➢ Length: 5
  - ➢ Knots: 0, 0, 0, 2, 4, 5, 5, 5
  - ➢ Points: 0, 9, 0, 9, 0

- C_2 has:
  - ➢ Degree: 3
  - ➢ Length: 5
  - ➢ Knots: 0, 0, 0, 0, 4, 5, 5, 5, 5
  - ➢ Points: 9, 10, 9, 10, 9





Product of B-spline Curves – Specific Interesting Interval [2,4]

- C_1 has:
  - Degree: 2
  - Length: 5
  - Knots: 0, 0, 0, 2, 4, 5, 5, 5
  - Continuities: 1 1
  - Points: 0, 9, 0, 9, 0
- C_2 has:
  - Degree: 3
  - Length: 5
  - Knots: 0, 0, 0, 0, 4, 5, 5, 5, 5
  - Continuities: 2
  - Points: 9, 10, 9, 10, 9
- C has:
  - Degree: 5
  - Knots: 0, 0, 0, 0, 0, 0, 2, 2, 2, 2, 4, 4, 4, 4, 5, 5, 5, 5, 5, 5
  - Continuities: 1 1
  - Conclusion: Length: 14

Product of B-spline Curves – Specific Interesting Interval [2,4]

- C has:
  - Degree: 5
  - Knots: 0, 0, 0, 0, 0, 0, 2, 2, 2, 2, 4, 4, 4, 4, 5, 5, 5, 5, 5, 5
  - Continuities: 1 1
  - Conclusion: Length: 14
- Evaluation points for interpolation: 2, 2.4, 2.8, 3.2, 3.6, 4.0
- $C_1 C_2$ values: 42.975, 29.7063936, 24.5369352, 27.4203648, 38.4767064, 58.08
- We have to solve a linear system of equations given by

$$c(t) = 16.\,x_5 - 32.\,x_6 + 40.\,x_6\,t - 1.\,x_5\,t^3 - 18.\,x_6\,t^2 + 3.5\,x_6\,t^3 + .062\,x_5\,t^4 - 16.\,x_5\,t + 6.\,x_5\,t^2 + 24.\,x_7 - .25\,x_6\,t^4 - 8.\,x_8 + 20.\,x_7\,t^2 - 4.5\,x_7\,t^3 + .38\,x_7\,t^4 - 36.\,x_7\,t - 9.\,x_8\,t^2 + 14.\,x_8\,t - .25\,x_8\,t^4 + 2.5\,x_8\,t^3 - .50\,x_9\,t^3 + 1.5\,x_9\,t^2 - 2.\,x_9\,t + x_9 + .062\,x_9\,t^4$$

Product of B-spline Curves – Specific Interesting Interval [2,4]

$$\begin{cases} c(2.0) = x_8 + 1.1x_5 + 4.8x_6 - 3x_7 = 42.975 \\ c(2.4) = 2x_9 + x_8 - 2x_7 - 2x_5 = 29.7063936 \\ c(2.8) = 24.5369352 \\ c(3.2) = 27.4203648 \\ c(3.6) = 38.4767064 \\ c(4.0) = 58.08 \end{cases}$$

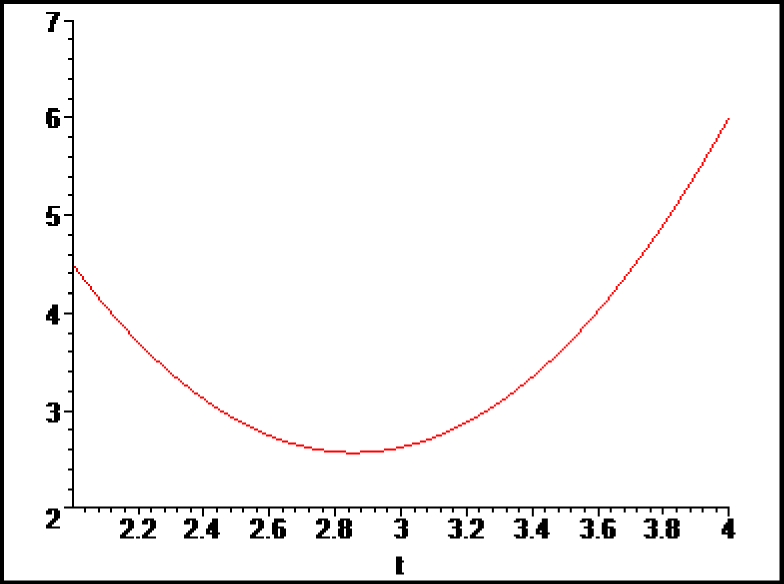
with

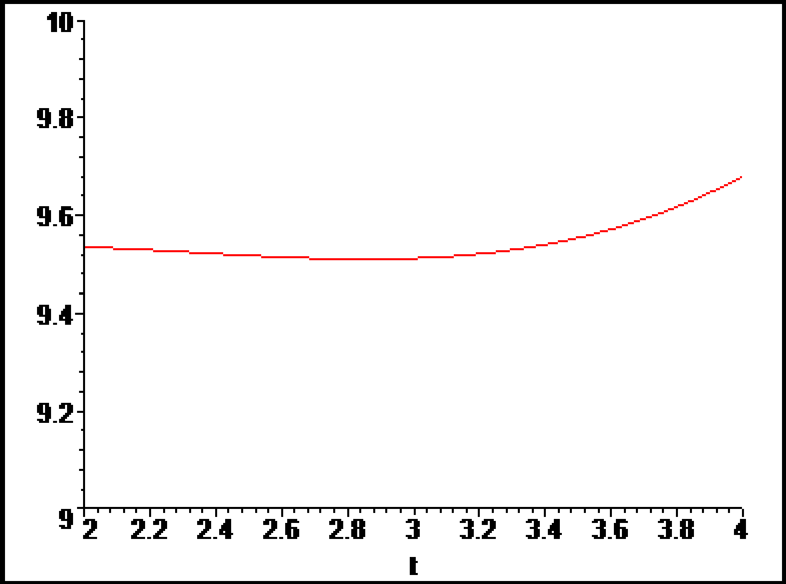
$$c(t) = x_4 B_{4,5}(t) + x_5 B_{5,5}(t) + x_6 B_{6,5}(t) + x_7 B_{7,5}(t) + x_8 B_{8,5}(t) + x_9 B_{9,5}(t)$$

Product of B-spline Curves – Specific Interesting Interval $[2,4]$

$c_1(t) = 24 - 15t + 2.6t^2$

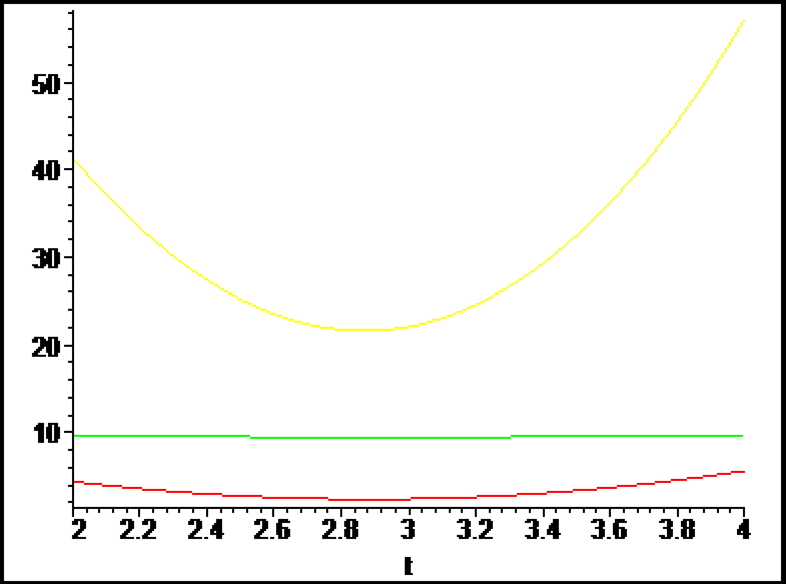
$c_2(t) = 9 + 0.75t - .34t^2 + 0.048t^3$





Product of B-spline Curves – Specific Interesting Interval $[2,4]$

$$c(t) = 216 - 117t + 3.99t^2 + 8.202t^3 - 1.604t^4 + 0.1248t^5$$
$$= 60B_{4,5}(t) + 26B_{5,5}(t) + 18B_{6,5}(t) + 21B_{7,5}(t) + 34B_{8,5}(t) + 70B_{9,5}(t)$$



Applications: Curvature Computation

- One knows:

$$k(t) = \frac{\|\beta' \times \beta''\|}{\|\beta'\|^3}$$

- Symbolic computations for β', β'' and $\beta' \times \beta''$
- Applications in
  - Morphing
  - Computer Vision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