236716
Computer Aided Geometric Design
HW 1

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Handed Out: May 19th, 2025 Due Date: May 20th, 2025 (12:30)

1. (30pt)
   - (15pt) Prove or provide a counter example: A parametric Bézier curve $C(t)$ is regular ($||C'(t)|| > 0$) if and only if all its control points different.
   - (15pt) Can a quadratic (degree 2) regular Bézier curve have a local curvature extremum? A local torsion extremum?
2. (30pt)
   - (15pt) Can you find two regular parametric curves, $c_1(t)$ and $c_2(t)$, $t \in [t_0, t_1]$, such that $\kappa_1(t) = \kappa_2(t)$ but c_1 and c_2 are not identical (up to rigid motion).
   - (15pt) Let $c(t)$, $t \in [t_0, t_1]$ be a regular parametric curve such that $\kappa(t) = const > 0$. Are all osculating (tangent) circles at all parameters $t \in [t_0, t_1]$ identical (same radius and orientation)? Prove or provide a counterexample.
3. (20pt) Prove or Dispute: A degree n Bézier curve $C(t)$ might be along a line, while (some of) the control points of $C(t)$ are not on the line.

4. (20pt) Propose an algorithm to compute the definite integral of the product of two Bézier basis functions $\theta_{i,n}(t)\theta_{j,m}(t)$:
$$\int_{t=t_1}^{t_2} \theta_{i,n}(t)\theta_{j,m}(t)dt.$$
An approximation solution will not be acceptable.