Computer Aided Geometric Design
HW 2
Instructor: Gershon Elber
T.A.: Boris van Sosin

Handed Out: July 7th, 2025   Due Date: July 8, 2025 (12:30)

1. (30 pt) Prove or dispute:
   (a) All the control points of a B-spline curve are on some B-spline surface. Hence, all the curve must be contained in the surface.
   (b) All the control points of a linear B-spline curve are on some B-spline surface. Hence, all the curve must be contained in the surface.
   (c) All the control points of a linear B-spline curve are on some bi-linear B-spline surface. Hence, all the curve must be contained in the surface.
   (d) A 3D bilinear Bézier surface must be contained in the tetrahedron defined by its 4 corners.

2. (20 pt) A parabolic point (curve) is a point (curve) on some surface, in which the Gaussian curvature is zero - $K = 0$.
   (a) What are the parabolic curves of a torus, if any?
   (b) What are the parabolic curves of a surface of revolution, if any?

3. (20 pt) Consider B, an axis-parallel bounding box of a B-spline surface S, with open-end conditions, computed from the control mesh of S.
   (a) S is C^1 continuous. Is B a tight axis-parallel bounding box of S?
   (b) S is only C^0 continuous. Is B a tight axis-parallel bounding box of S?
   (c) If either (a) or (b) or both are not tight, can you propose a solution to devised a tight axis-parallel bounding box for S? A hint: where can the global extrema of S occur?

4. (30 pt) Let $S(u, v)$ be a polynomial Bézier surface in R^3 with orders (m, n), and $c(t)$ be a piece-wise polynomial B-spline curve in R^2 of order q.
   (a) What conditions must be imposed on $c(t)$ so that the function composition $S(c(t))$ is well-defined?
   (b) Write the formula for computing $S(c(t))$ as symbolic operations on scalar B-spline curves and the (u, v) parameters of S. Leaving the answer as addition/product/etc. of scalar B-spline curves is fine.
   (c) Let $S(u, v)$ be a bi-cubic Bézier surface, and $c(t)$ a quadratic B-spline curve with the knot vector $0, 0, 0, 1, 1, 2, 3, 3, 3$. What are the polynomial degree and knot vector of $S(c(t))$?